CONTOH SKEMA JAWAPAN

Question 2.

If $a_n = \frac{2^3-1}{2^3+1}\cdot\frac{3^3-1}{3^3+1}\cdots\frac{n^3-1}{n^3+1}$, find $\lim_{n\to\infty} a_n$. **(10 Marks)**

Solution

	Mark
$x^3 - y^3 = (x-y)(x^2+xy+y^2)$ and $x^3 + y^3 = (x+y)(x^2-xy+y^2)$	K1 K1
$a_n = \frac{2^3-1}{2^3+1}\cdot\frac{3^3-1}{3^3+1}\cdots\frac{n^3-1}{n^3+1}$ $= \prod_{k=2}^{n}\frac{(k-1)(k^2+k+1)}{(k+1)(k^2-k+1)}$ $= \prod_{k=2}^{n}\frac{k-1}{k+1}\prod_{k=2}^{n}\frac{k^2+k+1}{k^2-k+1}.$	K1 K1
$\prod_{k=2}^{n}\frac{k-1}{k+1} = \frac{1}{3}\cdot\frac{2}{4}\cdot\frac{3}{5}\cdot\frac{4}{6}\cdots\frac{n-3}{n-1}\cdot\frac{n-2}{n}\cdot\frac{n-1}{n+1} = \frac{2}{n(n+1)}$	K1,J1
$\prod_{k=2}^{n}\frac{k^2+k+1}{k^2-k+1} = \prod_{k=2}^{n}\frac{k^2+k+1}{(k-1)^2+(k-1)+1} = \frac{n^2+n+1}{3}$	K1,J1
$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(\frac{2}{n(n+1)}\right)\left(\frac{n^2+n+1}{3}\right) = \frac{2}{3}$	K1, J1
	10 marks

K=Kaedah, J=Jawapan